## The geometry of successive cyclicity

1. Introduction. The idea that syntactic objects are best modelled set-theoretically stretches back to their treatment as sets of strings in Chomsky (1955). Most recently, the operation Merge recursively produces set theoretic objects. This leads to well-known theoretical problems with labelling, and copies vs repetitions, for which extra sub-theories are required. A recent alternative takes syntactic objects to be mereological rather than set-theoretic (Adger 2023) and proposes that they are built via an operation $\operatorname{Subjoin}(\alpha, \beta)$ which makes $\alpha$ a (proper) part of $\beta$, where '(proper) part of' $(<)$ is to be understood as the irreflexive transitive (asymmetric) relation of mereology (Coitnoir and Varzi 2022). First subjunction to an object creates what is called a 1-part relation that is syntactically interpreted as extended projection (Grimshaw 1990) while second subjunction to that object creates a 2-part relation interpreted as specifierhood. For example, in the unergative example in Fig 1, V subjoins to v, becoming part of it and relating to it via the extended projection relation, and $N$ Subjoins to $D$ in the same way. $D$ then subjoins to v ; this second subjunction to v is interpreted as making D a specifier of v . The complex object v then subjoins to T . What is derived is a 'telescoped' representation (as in Brody 2000), where the extended projections are lexicalized via a spanning mechanism (Svenonius 2016).

Fig. 1 T


Fig. 2 d


Fig. 3 d


Fig. 4 T*


This theory does not require special subtheories for labelling or to distinguish copies from repetitions. Subjunction does not alter labels (the label of the target of subjunction remains the same) and the system provides multidominance (or rather multiparthood) structures, which structurally distinguish between copies and repetitions. For example, in Fig 2, $b$ has subjoined to $d$ (a second subjunction to $d$ ), while in Fig $\mathbf{3}$ a second $b$ has subjoined to $d$. These representations are distinct enough so that the PF and semantic systems can treat them differently, unlike in the standard set-theoretic system. No extra sub-theories are required for labelling or for copies.
2. Locality. Locality in this system is a geometrical constraint on subjunction (cf. Kayne's 1983 Connectedness), allowing at most one angle in the resulting structure (Angular Locality):
(1) $\gamma$ can subjoin to $\beta$ only if there is an $\alpha$ s.t. $\gamma$ is an n-part of $\alpha$ and $\alpha$ is a 1-part of $\beta$

In Fig. 4, D can subjoin to $T$ (since $D$ is 2-part of $v$ which is a 1-part of T), but $N$, for example, cannot ( 2 angles). This theory of Locality excludes the mereological equivalent of sidewards/parallel movement which would have to involve subjunction of an object to a target which is entirely disconnected from it (since the target is not in the same structure as the subjoining object, it cannot satisfy Angular Locality). It also excludes super-local movement, since if, for example, v were to subjoin to T in Fig 1, the equivalent of moving a complement of a head to its specifier, there would need to be some object that $v$ is part of (which can only be $T$ ) and that object would have to be a 1-part of T. However, parthood in this system is irreflexive ( T is not a part of T). It follows that super-local subjunction is ruled out, deriving the kind of Anti-Locality argued for by Abels (2003). The system ensures that the single object D in Fig 4 is spelled out high rather than low in the structure via a second-order EPP-type feature, which I represent as *. Note that only one * for an object will be possible in a single extended projection, or else the PF systems will have conflicting instructions as to how to spell out the object.
3. Successive Cyclicity. Angular Locality, however, seems too strict, since it will rule out nonlocal dependencies from inside a specifier to outside that specifier, blocking long distance movement. If, for example, $u$ in Fig 5 is a clause which is an object of a higher V, Angular Locality will block subjunction of $z$ outside of the clause. However, the transitivity of parthood provides a loophole. If $z$ subjoins to $u$ as in Fig 6, then, because parthood is transitive, $z$ is a part of $e$, and since $e$ is a 1-part of $y$, Angular Locality allows subjunction of $z$ to $y$ :

Fig. 5


Fig. 6


Fig. 7


Fig. 8 C*


The upshot is that subjunction out of a specifier, requires prior subjunction to that specifier. This derives, without phases etc., successive cyclicity of non-local 'movement' dependencies. It also provides answers to four theoretical puzzles in locality theory that do not have satisfactory answers in classical approaches: why there are locality domains at all (Angular Locality disallows syntactic relationships with multiple 'angles' in the structure); why those domains are what they are (the domains are specifiers because these make an 'angle'); why locality domains have 'escape hatches' (this is just a consequence in the mereological system of the transitivity of parthood) and why those exceptions are what they are (again, this is a consequence of transitivity). Classical approaches do not lead us to expect any constituent to be a locality domain, they force us to stipulate the existence of phases and to stipulate which categories are phasal. In addition they require a stipulation that phases have edges, and further stipulations about what those edges are.
3. Empirical Consequences. The system creates the equivalent of 'parallel movement chains' (as in Chomsky 2008) for successive cyclic movement. Take the Scottish Gaelic example in (2):
(2) Ancat a thuirt Daibhidh a/*gu(m) bhuaileas Calum $\Delta$
the cat REL say.PST David REL/*sc that hit.FUT.REL Calum
'The cat that David said Calum will hit.'
The relativized object is $\Delta$, but $\Delta$ is part not just of the lowest $v$, but also of the intermediate and topmost Cs (as in Fig 7). Subjunction to the intermediate C is required to make $\Delta$ a 2-part of the topmost extended projection, but serves no semantic purpose; it does, however, in Gaelic, condition the form of the C in (2) (Adger and Ramchand 2005). It is, then, a coincidence that the Cs share the same form. The system predicts that the topmost C should be able to be distinct in form, which is what we find in e.g. Kîtharaka (Muriungi 2005). An alternative derivation that will also converge would be that the intermediate C , as opposed to $\Delta$, subjoins to the topmost C , which also brings about the requisite transitivity, making $\Delta$ a 2-part of $\mathrm{C}^{*}$, and derives the well known CP pied-piping strategy of, e.g. Basque (Ortiz de Urbina 1989). Finally, one might ask how cases where one object that has undergone successive subjunction could be spelled out in multiple positions (as in Afrikaans, de Plessis 1977). In fact, exactly in such cases, where there are distinct extended projections, we can have multiple * features without issuing conflicting instructions to the PF systems, so that the same object will be spelled out at each C*, as in Fig 7 above but with * also on the intermediate C .

